WNE Linear Algebra Resit Exam
Series A

2 March 2019

Please use separate sheets for different problems. Please provide the following data on each sheet

- **name, surname and your student number,**
- **number of your group,**
- **number of the corresponding problem and the series.**

Problem 1.
Let $V = \mathrm{lin}((1,1,2,-1),(-2,1,5,8),(4,1,-1,-10))$ be a subspace of $\mathbb{R}^4$.

a) find a basis $\mathcal{A}$ of V and the dimension of V,
b) find all $t \in \mathbb{R}$ such that $(2,1,1,t) \in V$.

Problem 2.
Let $V \subset \mathbb{R}^4$ be a subspace given by the homogeneous system of linear equations

$$\begin{cases} x_1 + 2x_2 - 3x_3 + 11x_4 = 0 \\ -2x_1 - 4x_2 - x_3 + 6x_4 = 0 \\ 5x_1 + 10x_2 - 5x_4 = 0 \end{cases}$$

a) which of the following sequences of vectors are bases of V?
   i) $((1,2,-3,11))$,
   ii) $((1,2,-3,11),(-2,-4,-1,6))$,
   iii) $((-2,1,0,0),(1,0,4,1))$,
   iv) $((3,-1,4,1),(1,0,4,1))$,
   v) $((-2,1,0,0),(4,-2,0,0))$.
   Give reasons for your answers.
b) find coordinates of $w = (3,-2,-4,-1)$ relative to one of the basis from the part a).

Problem 3.
Let

$$A = \begin{bmatrix} -2 & 3 \\ -6 & 7 \end{bmatrix}.$$

a) does there exists matrix $C \in M(2 \times 2; \mathbb{R})$ such that

$$C^{-1}AC = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

for some $a, b \in \mathbb{R}$ such that $a < b$? If it does, give an example of such matrix C.
b) compute A^{200}.

Problem 4.
Let $\mathcal{A} = ((1,2),(1,3))$, $\mathcal{B} = ((0,1),(1,2))$ be ordered bases of $\mathbb{R}^2$. Let $\varphi\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformations given by the matrix

$$M(\varphi)_{\mathcal{A}}^{\mathcal{B}} = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}.$$

Let $\psi\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformations given by the matrix

$$M(\psi)_{\mathcal{A}}^{st} = \begin{bmatrix} 0 & -1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}.$$

a) find the formula of φ,
b) find the matrix $M(\psi \circ \varphi)_{\mathcal{B}}^{st}$.

Problem 5.
Let

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 5 & 2 \\ -4 & t & 5 \end{bmatrix}.$$

a) for which $t \in \mathbb{R}$ is matrix $A^4 A^\intercal$ invertible?
b) for which $t \in \mathbb{R}$ is the entry in the first row and the second column of A^{-1} equal to 10?

Problem 6.
Let $V = \operatorname{lin}((1, 2, -1), (3, 8, -5), (2, 1, 1))$ be a subspace of $\mathbb{R}^3$.
a) find an orthonormal basis of $V^\perp$,
b) compute the orthogonal projection of $w = (1, 0, 0)$ onto V.

Problem 7.
Let M be the affine plane in $\mathbb{R}^3$ passing through the point $Q = (1, 1, 7)$ which is parallel to the subspace

$$V = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid 2x_1 - x_3 = 0\}.$$

a) find a parametrization of the line L passing through point $P = (1, 0, 2)$ perpendicular to M,
b) find an equation describing M and compute the affine orthogonal projection of the point P onto M.

Problem 8.
Consider the following linear programming problem $-3x_2 - 2x_4 \to \min$ in the standard form with constraints

$$\begin{cases} x_1 \;+ \qquad\qquad + \;\; x_4 = 1 \\ \qquad x_2 \;+ \qquad - \; 2x_4 = 2 \\ \qquad\qquad x_3 \;+ \;\; x_4 = 3 \end{cases} \quad \text{and } x_i \geqslant 0 \text{ for } i = 1, \ldots, 4$$

a) which of the sets $\mathcal{B}_1 = \{1, 2, 3\}, \mathcal{B}_2 = \{1, 3, 4\}$ is basic feasible? Write the corresponding feasible solution.
b) solve the linear programming problem using simplex method.